ECON 4930 Spring 2011
Electricity Economics
Lecture 13

Lecturer:

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The key theme: drivers of price changes

- Point of departure: water as a natural resource in limited supply
  - All prices equal, Hotelling's rule (Chapter 2)
- But: The price of electricity varies over both day and season
- We must come up with explanations
- NB! price variation in the market may not be optimal

Constraints as price drivers (Chapter 3)

- Reservoir constraint
  - Threat of overflow
  - Running empty
- Production constraint
  - Pipes, turbines, generator, transmission
- Environmental constraints
  - Ramping up and down
  - Minimum production (max covered above)

The basic model (Chapters 1,3)

- The social optimisation problem

$$\max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz$$

subject to

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$R_t, e_t^H \geq 0 \ , \ t = 1,..,T$$

$$T, w_t, R_o, \overline{R} \text{ given, } R_T \text{ free}$$

The Lagrangian

$$L = \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

The first-order conditions

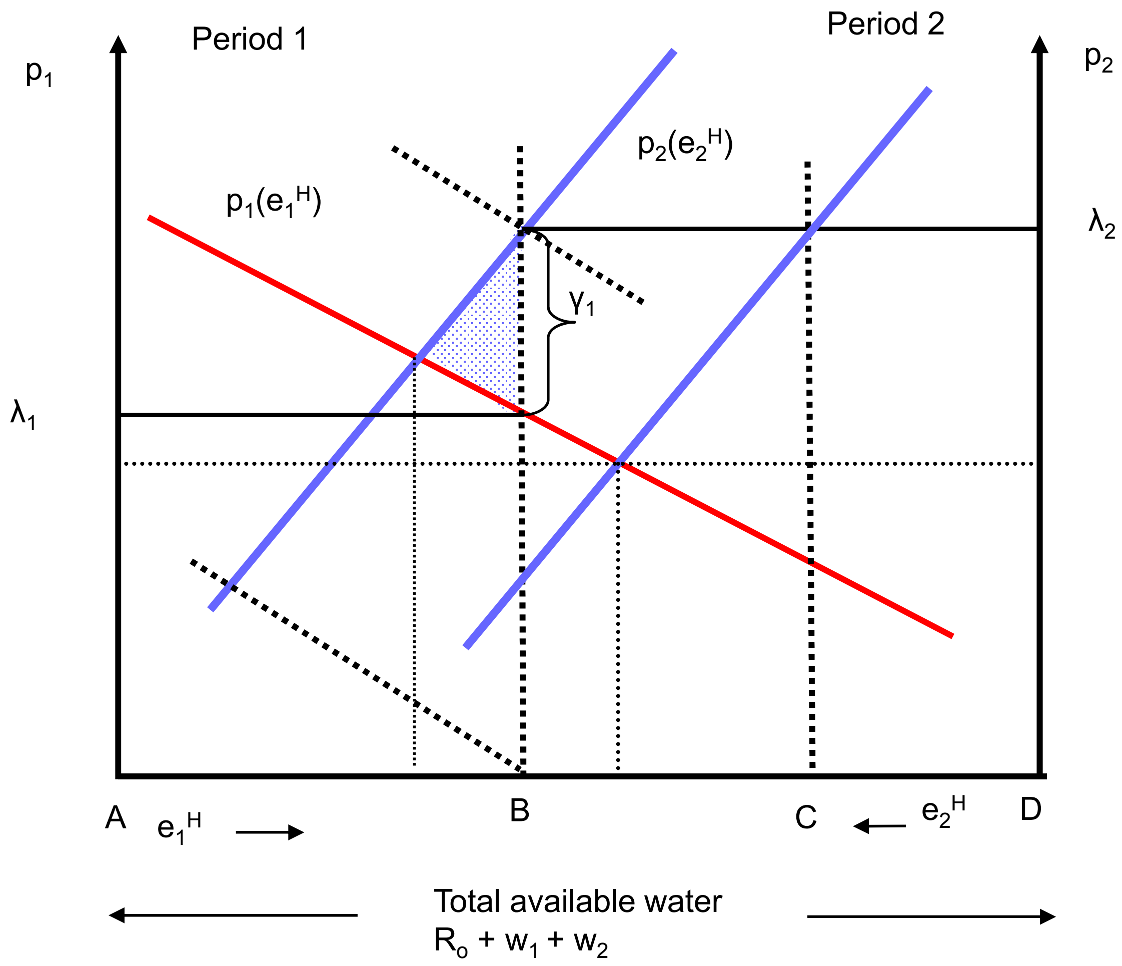
$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

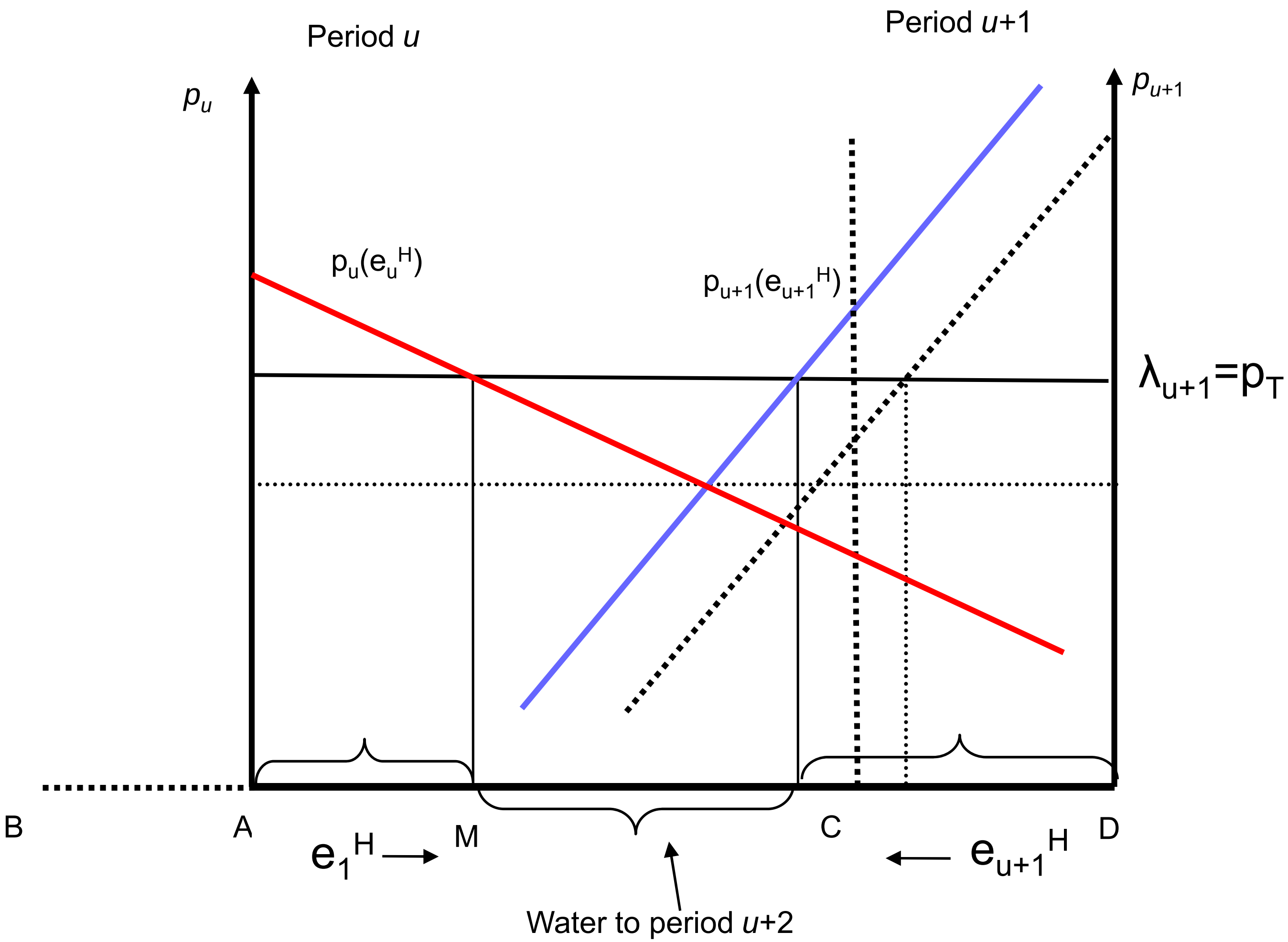
$$\lambda_t \geq 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \geq 0 \ (= 0 \text{ for } R_t < \overline{R})$$

Threat of overflow and empty reservoir



In between empty and full



Production constraint

- Production constraint

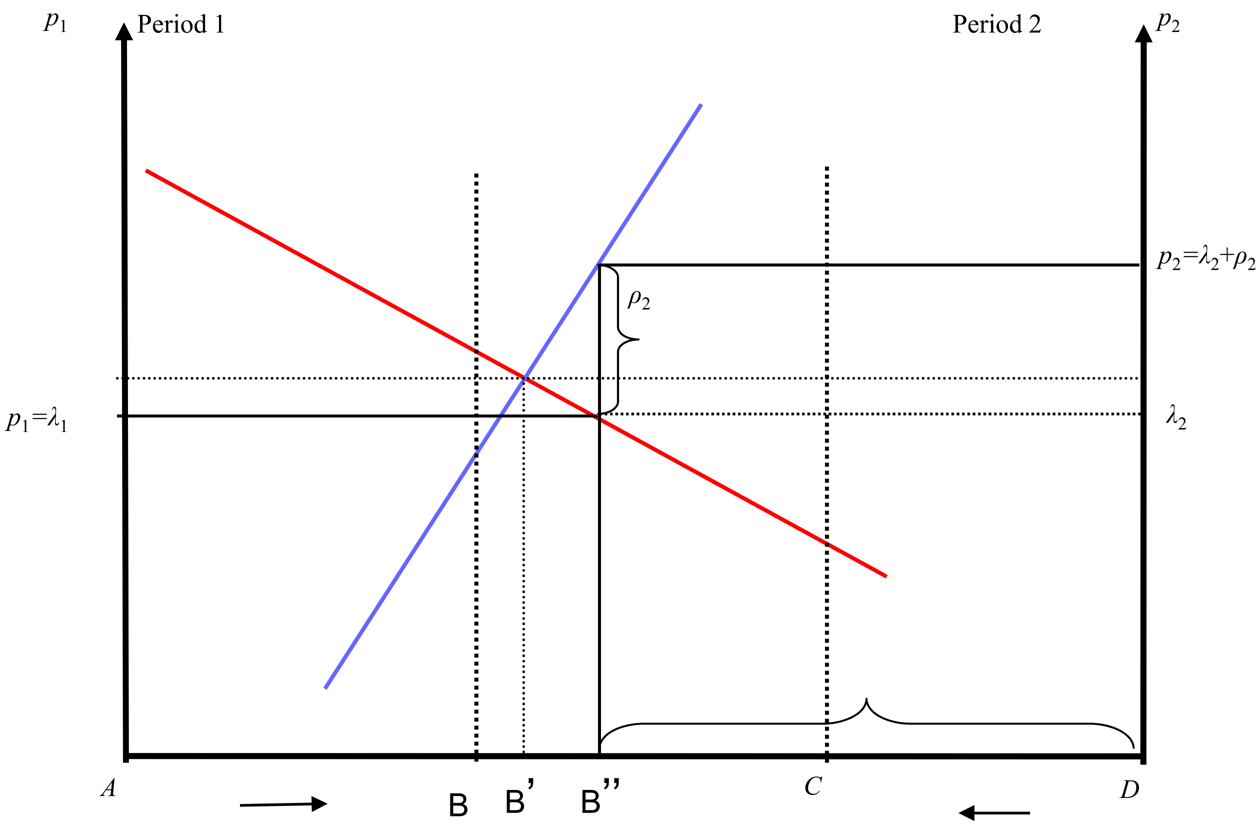
$$e_t^H \leq \overline{e}^H$$

- New first-order condition

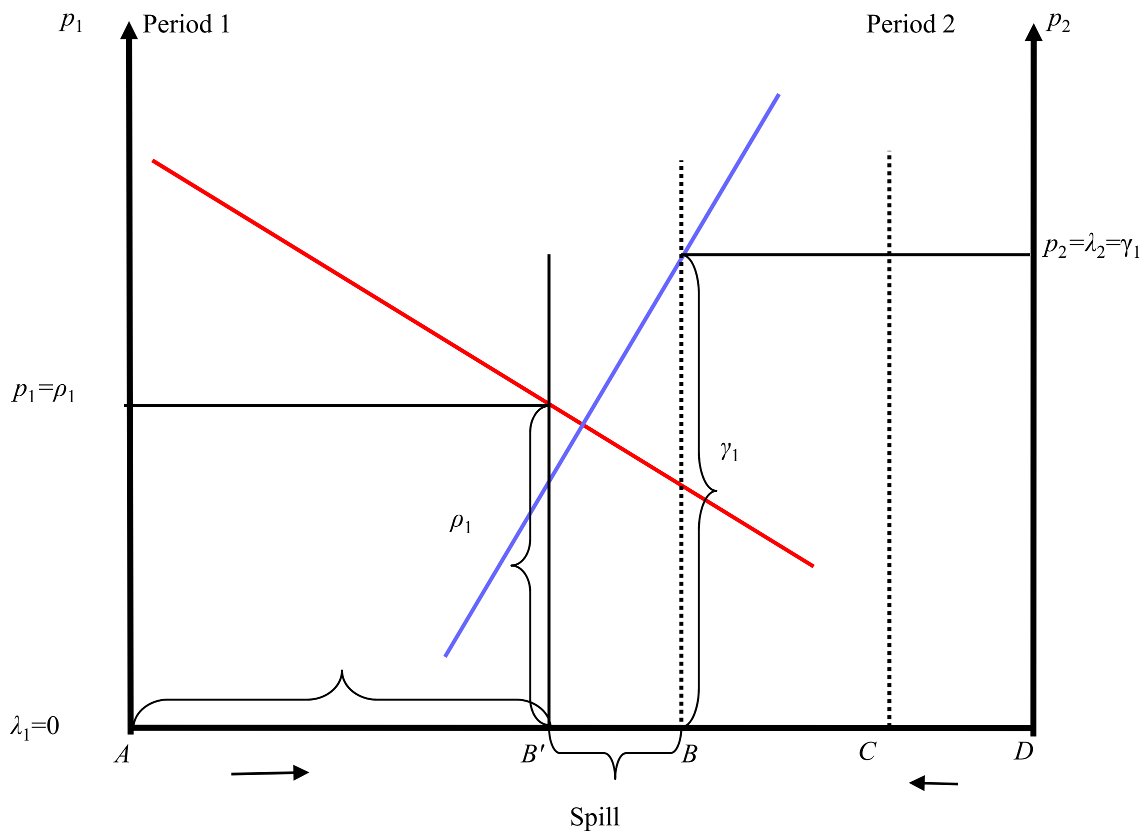
$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t - \rho_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\rho_t \geq 0 (= 0 \text{ for } e_t^H < \overline{e}^H), \qquad t = 1,..,T$$

Production constraint:
constraint in period 2: peak demand



Production constraint:
constraint in period 1: may have overflow



Unregulated generation

- Run-of-the-river
  - Full reservoirs turn a power station into a run-of-the-river station
- Windmills
  - Range of fluctuation in production large
- Solar
  - Nights, overcast

New relations of unregulated generation

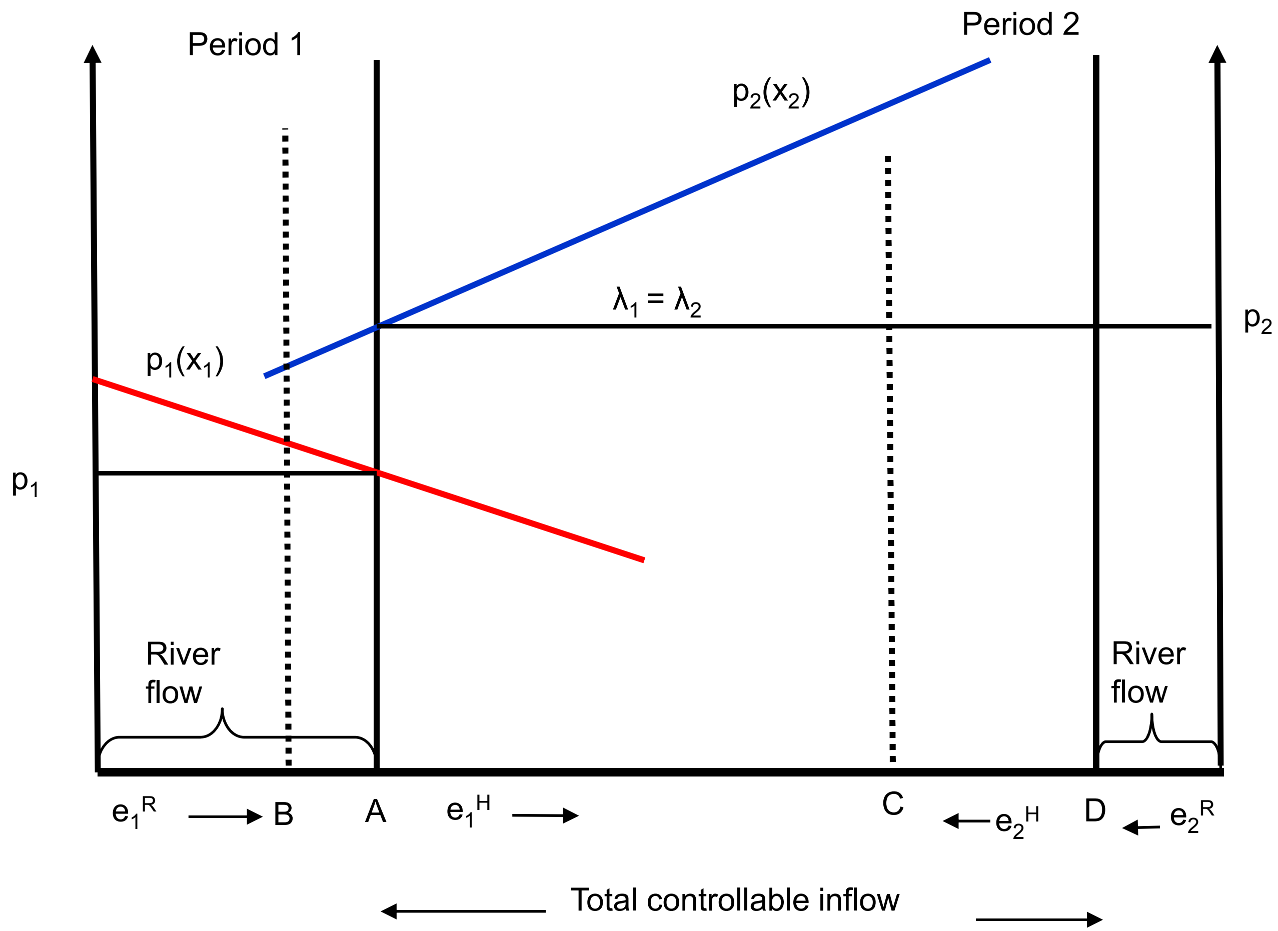
- Energy balance

$$x_t = e_t^H + e_t^I$$

- First-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^I) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

Must take: Run-of-the-river and wind power



Multiple producers (Chapter 4)

- Introduce model with N producers
  - Social optimisation problem

$$\max \sum_{t=1}^{T} \int_{z=0}^{x_t} p_t(z)\,dz$$

subject to

$$x_t = \sum_{j=1}^{N} e_{jt}^H$$

$$R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H$$

$$R_{jt} \le \overline{R}_j$$

$$R_{jt}, x_t, e_{jt}^H \ge 0$$

$$T, w_{jt}, R_{jo}, \overline{R}_j \text{ given}, R_{jT} \text{ free}, \; j = 1,..,N, \; t = 1,..,T$$

Hveding's conjecture (Chapter 4)

- Assume independent hydropower plants with one limited reservoir each, and perfect manoeuvrability of reservoirs, but plant-specific inflows
- The plants can be regarded as a single aggregate plant and the reservoirs can be regarded as a single aggregate reservoir when finding the social optimal solution for operating the hydropower system.
  - If overflow, then all reservoirs overflow at the same time, if empty then all reservoirs are emptied at the same time

Price fluctuation when hydro interacts with thermal (Chapter 5)

- Thermal sector is used according to merit order ranking of marginal cost; sectoral cost function
- General rule: price equals water value equals marginal cost of thermal
- Typical result: price variations less than in a pure hydro system

New relations with thermal

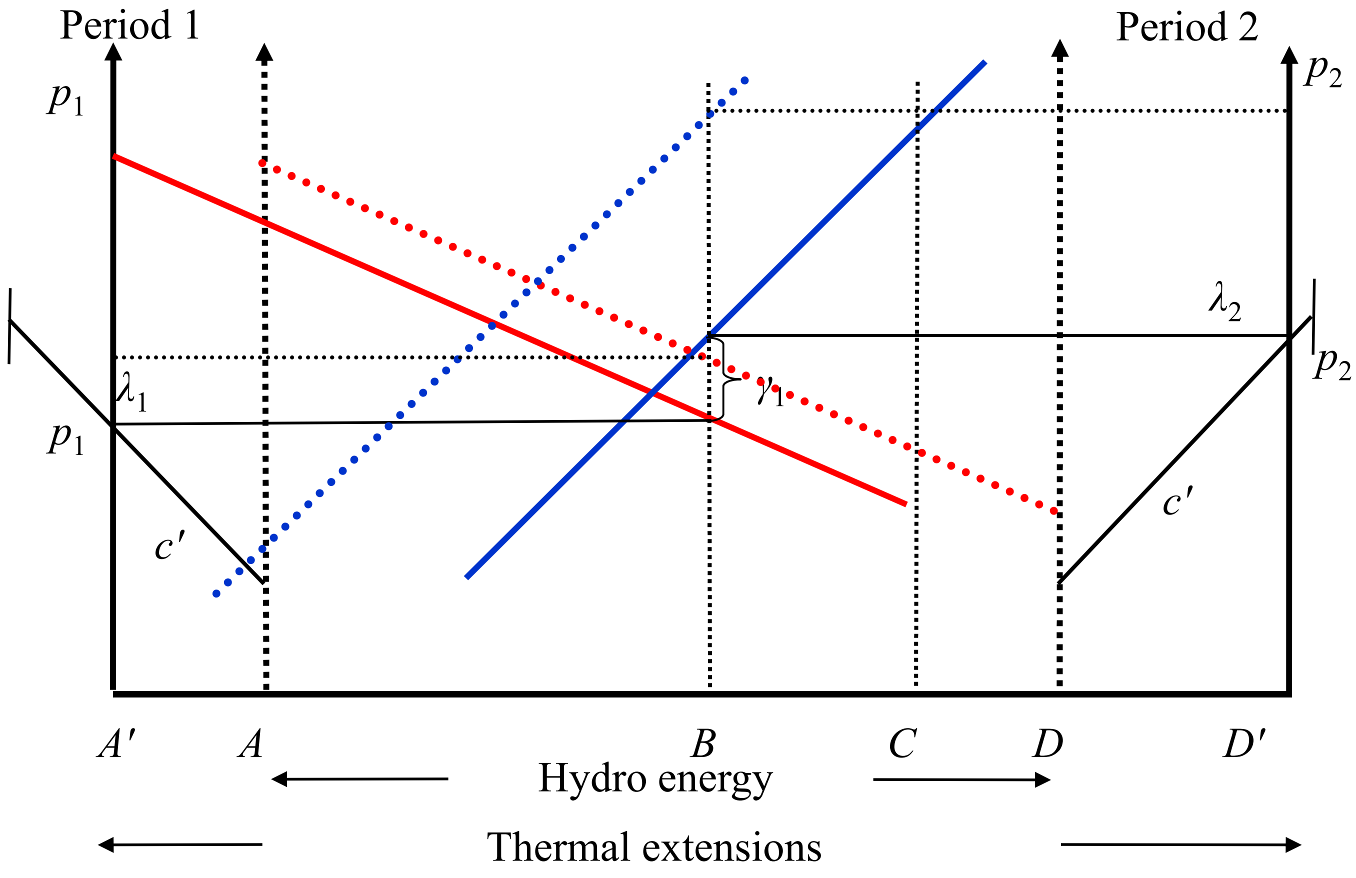
- Cost function included in the objective function: $c(e_t^{Th})$
- Energy balance: $x_t = e_t^H + e_t^{Th}$
- First-order condition

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) - \lambda \leq 0 \; (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \leq 0 \; (= 0 \text{ for } e_t^{Th} > 0)$$

$$\theta_t \geq 0 \; (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})$$

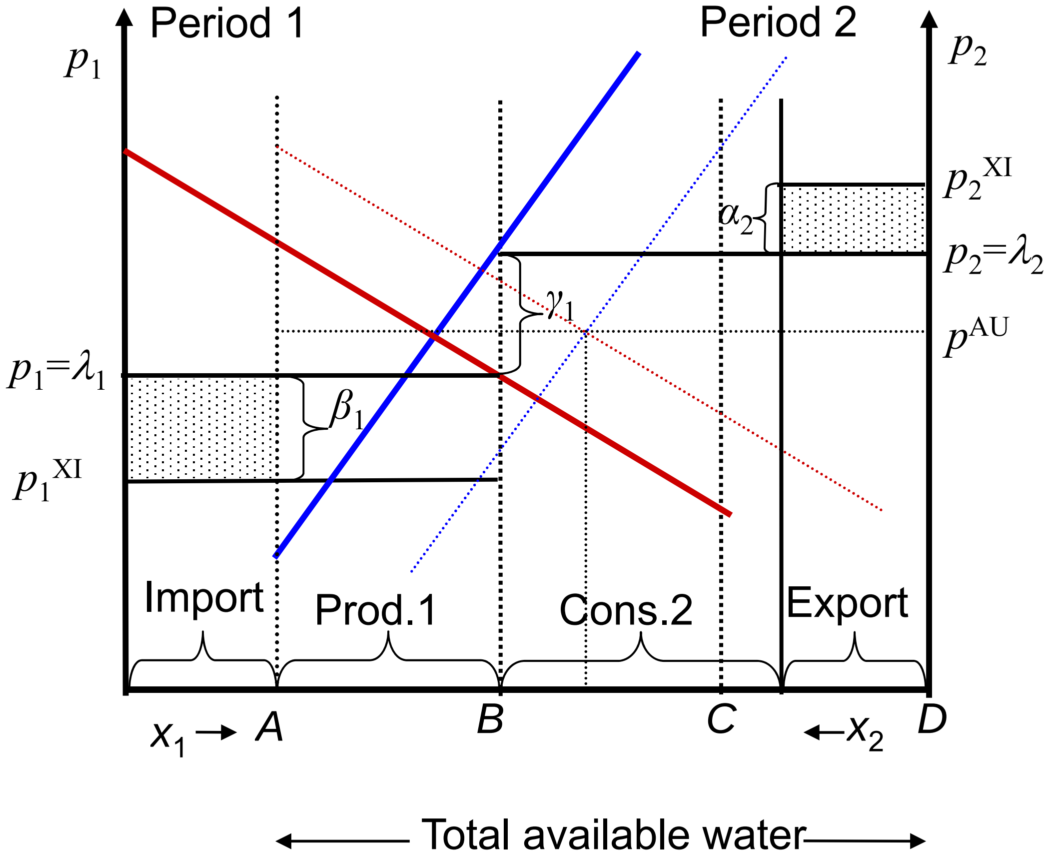
Bathtub diagram with thermal and hydro with reservoir constraint



The impact of trade on prices (Chapter 6)

- Unlimited trade
  - External exogenous prices determine the regional prices
- Limited trade due to transmission
  - Region may have different prices if trade is limited
  - If import-constrained higher regional price
  - If export-constrained lower regional price
- Endogenous trade prices
  - Equal prices without constraints on interconnectors
  - Different prices with constraints on interconnectors

Impact of trade on prices



The impact of transmission on prices (Chapter 7)

- Transmission: connecting all generator nodes and consumption nodes by lines
- Fundamental physics: loss of energy on lines (Ohm’s law)
  - Implication: nodal prices as the optimal price structure; higher consumer price the higher the loss
  - Implications for use of hydro over seasons
- Limited capacity of lines and congestion
  - Thermal capacity (Ohm’s law)
  - More general resistance; impedance when AC
  - Loop flows and other electric mysteries with AC

Transmission relations two nodes

- Energy balance and loss

$$x_t + e_t^L = x_t + e_t^L(x_t) = e_t^H, \; t = 1,2$$

- New first-order conditions

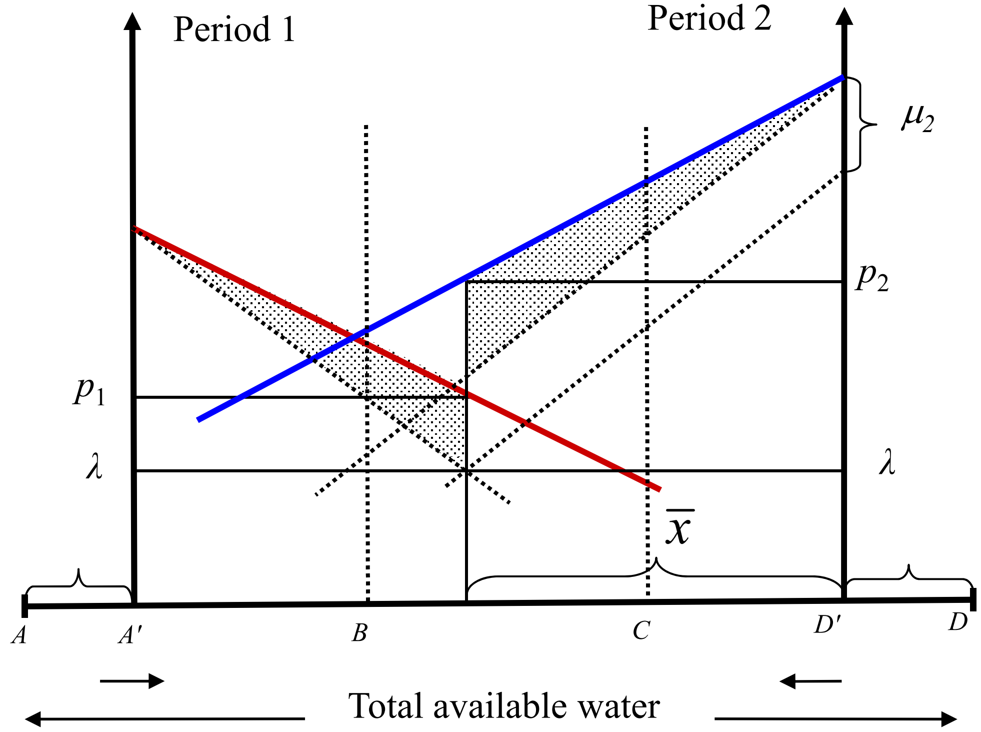
$$\frac{\partial L}{\partial x_t} = p_t(x_t) - \tau_t - \tau_t \frac{\partial e_t^L}{\partial x_t} - \mu_t \leq 0 \; (= 0 \text{ for } x_t > 0)$$

$$\frac{\partial L}{\partial e_t^H} = -\lambda_t + \tau_t \leq 0 \; (= 0 \text{ for } e_t^H > 0)$$

$$\mu_t \geq 0 (= 0 \text{ for } x_t < \overline{x}) \, , t = 1,2$$

- Nodal price: $p_t(x_t) - \lambda_t = \lambda_t \frac{\partial e_t^L}{\partial x_t} + \mu_t \, , t = 1,2$

Bathtub with loss and congestion



Uncertainty as price driver (Chapter 9)

- Assuming all information for the present period to be known, price for the next period will be expected price
  - The construction and use of expected water value table, role of constraints
- When time progresses there will be a continuous update of expected prices
- Realised price will typically differ from expected price, implying fluctuating price independent of binding constraints

Uncertainty model, two periods

- Social optimisation problem

$$\max_{e_1^H} \left[\int_{z=0}^{e_1^H} p_1(z)\,dz + E\left\{ \int_{z=0}^{R_o + w_1 + w_2 - e_1^H} p_2(z)\,dz \right\} \right]$$

subject to

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

$$R_1 \in \left[0, \overline{R} \right]$$

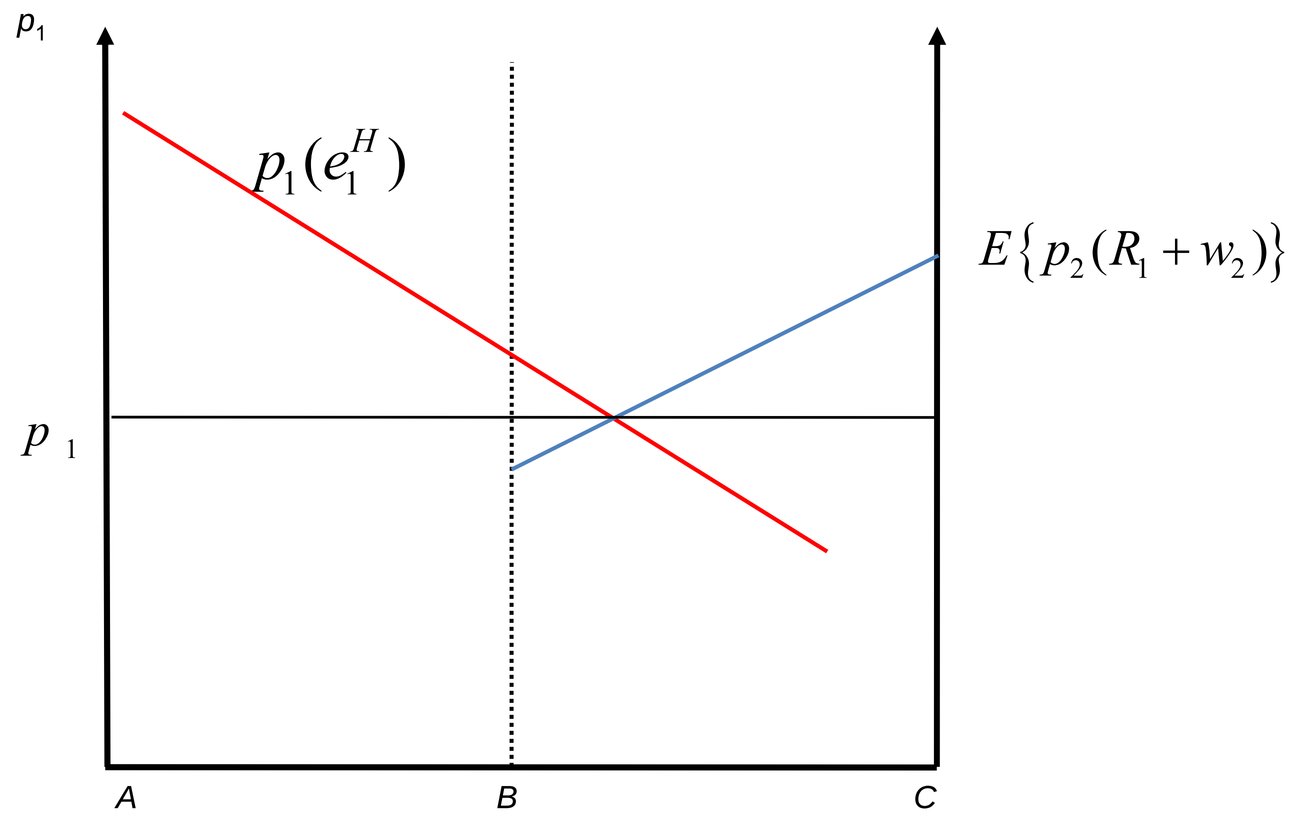
- First-order conditions
  - Interior solution

$$p_1(e_1^H) - E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\} = 0 \Rightarrow$$

$$p_1(e_1^H) = E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\} = E\left\{p_2(R_1 + w_2)\right\}$$

  - Expected price table: $E\left\{p_2(R_1 + w_2)\right\}$

Illustration of uncertainty for period 2 making decision in period 1



Market power as price driver: Monopoly (Chapter 8)

- Shifting of water from relatively inelastic periods to more elastic periods
  - Price will then go down in periods where more is produced and up in periods with less production
  - Possibility of spill
  - Possibility of no unique solution
  - Social price solution if reservoir constraint is binding

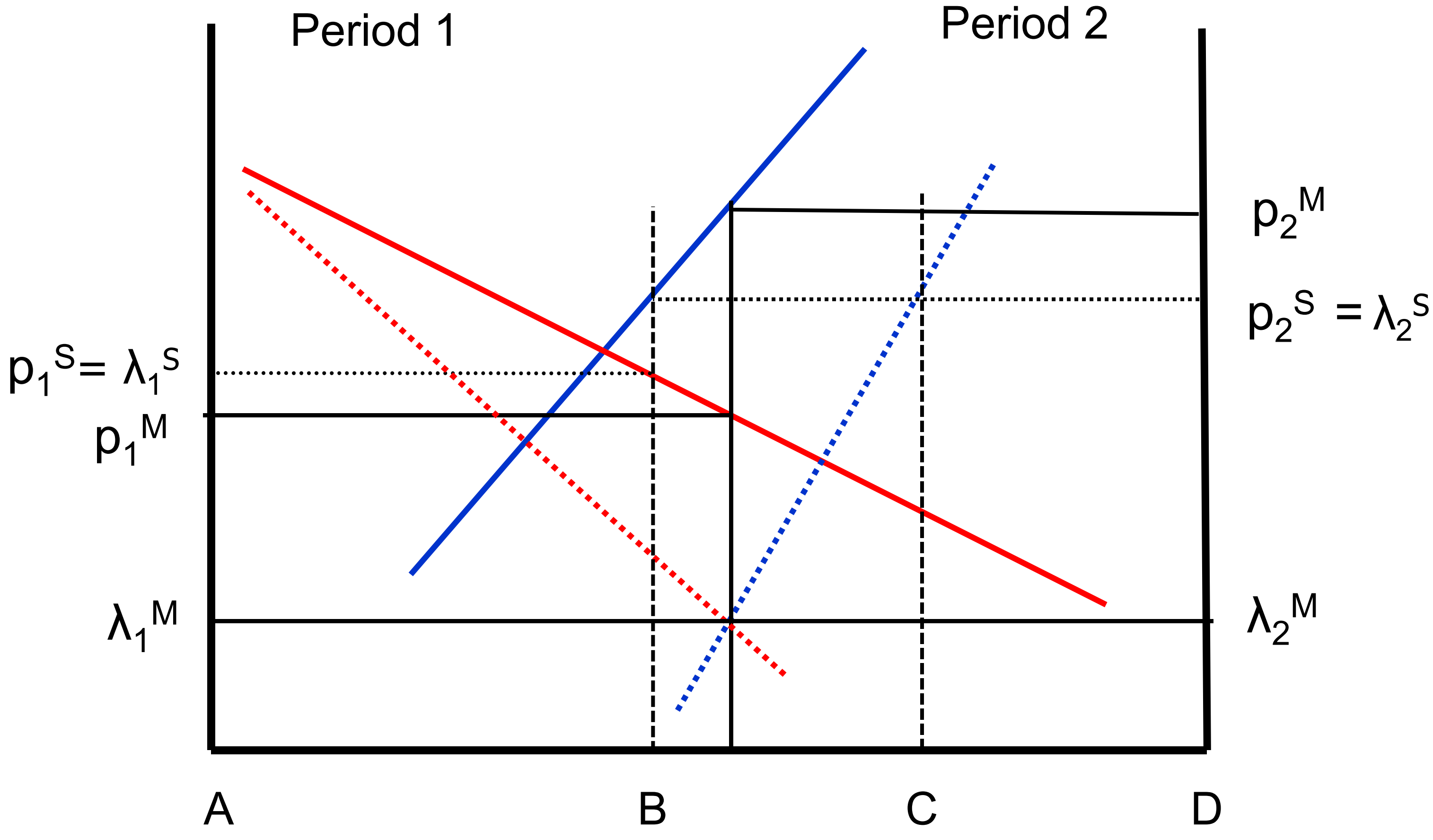
The monopoly model

- First-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0) \Rightarrow$$

$$p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t = p_t(e_t^H)(1 + \breve{\eta}_t) - \lambda_t = 0$$

Bathtub illustration of two periods



The competitive solution (Chapter 10)

- Second welfare theorem: any efficient allocation can be sustained by a competitive equilibrium
- Problems:
  - Electric externalities due to transmissions with loop flows, reactive power, etc.
  - Hydraulic externalities for plants along the same river system
  - Uncertainty and expectation formation

Investments (Chapter 10)

- Deregulation of electricity sector
  - Unbundling generation and transmission
- Investment in generation and investment in transmission made by independent organisations, but investments need coordination both over time and over space
- Use of shadow prices on capacities as marginal investment signals
  - Lumpy investment - indivisibilities